Sample question paper – Model answers *

AE 1301 Flight dynamics

B.E./B.Tech Degree examination,

November / December 2006.

Anna University.

Time: 3 hours **Maximum: 100 marks**

Answer ALL questions

Part A- (10 x 2 = 20 marks)

1. What causes induced drag?

Answer:

Consider a wing which is producing lift. A wing has a finite span. Since, there cannot be a pressure discontinuity in subsonic flow, the pressure at the wing tips would be same on the upper and lower sides. However, near the wing root the pressure on the upper surface would be, on an average, lower than the atmospheric pressure and that on the lower surface, on an average, would be higher than the atmospheric pressure. Because of the pressure differences between the root and the tip, there will be a flow from tip to the root on the upper surface and from root to tip on the lower surface. Thus, the streams from the upper and lower sides of the wing would meet, at the trailing edge, at an angle. This ultimately results in

a system of trailing vertices. These vertices induce an angle on the flow called

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***** In the section entitled “Sample question paper - hints for solution” , some hints were given for locating the answers in the lecture materials on Flight Dynamics-I and II. Herein, model answers are given. These could be used as guidelines regarding the ways to extract and present the answers from the lecture materials.

induced angle, which tilts the aerodynamic force rearword. The component in the stream wise direction is the induced drag. It may be added that the induced drag coefficient (C_{Di}) is given by:

$$C_{Di} = \frac{C_L^2}{\pi A}(1+\delta)$$

Where A is wing aspect ratio and δ is a factor which depends on wing parameters like aspect ratio, taper ratio and sweep.

2. Plot the variation of power available with flight speed for a propeller powered airplane and indicate the effect of altitude on the curve.

Answer:

The following may be pointed out about the power output of a piston engine and the efficiency of a propeller.

(i) The BHP of a pistion engine at a given altitude and engine RPM, shows only a slight increase with flight speed. Hence, the engine output (BHP)can be assumed to be nearly constant with flight speed.

(ii) Due to decrease in the atmospheric density (ρ) with altitude, the BHP of the engine decreases with altitude and is approximately given as:

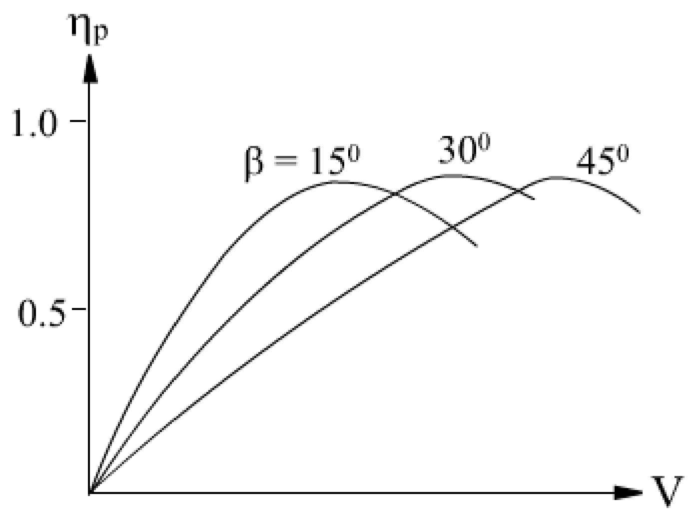
$$BHP = (BHP)_{s.l}\,(1.13\,\sigma - 0.13)$$

$$\sigma = \rho / \rho_{sl}$$

(iii) Thrust horse power (THP) is given as :

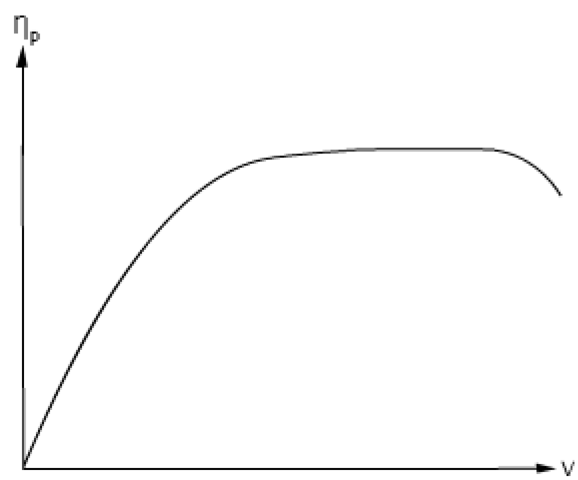
$THP = \eta_p\, BHP$, η_p = propeller efficiency

(iv) The propeller efficiency (η_p) depends on the flight velocity (V) and the pitch setting (β) of the propeller.Typical variations of η_p with V and β are shown in the figure below.



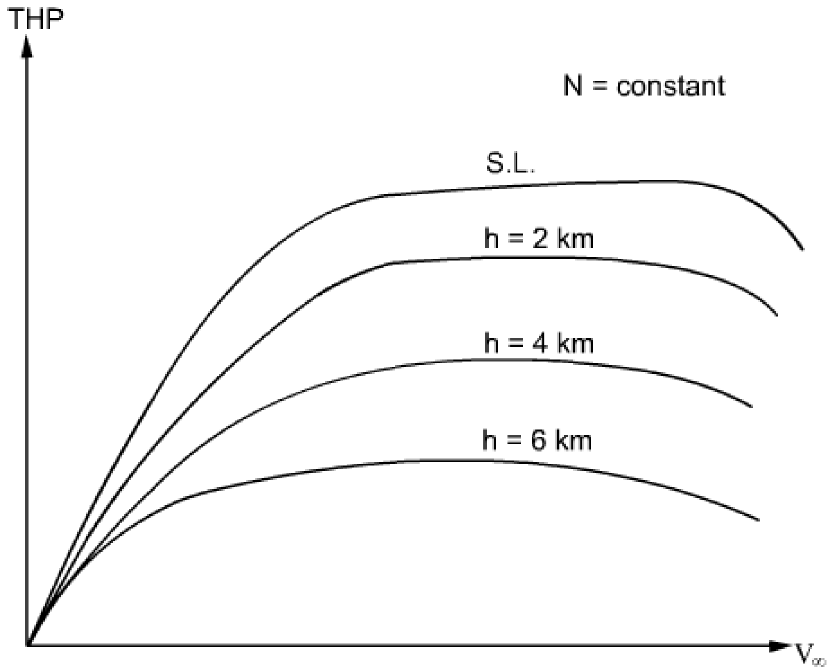
Schematic variation of η_p vs V with β as parameter

For a variable pitch propeller the variation would be is as shown below.



Variation of η_p vs V in a case with a variable pitch propeller

Keeping in view these aspects, the variations of THP with velocity and altitude are schematically presented in the figure below.

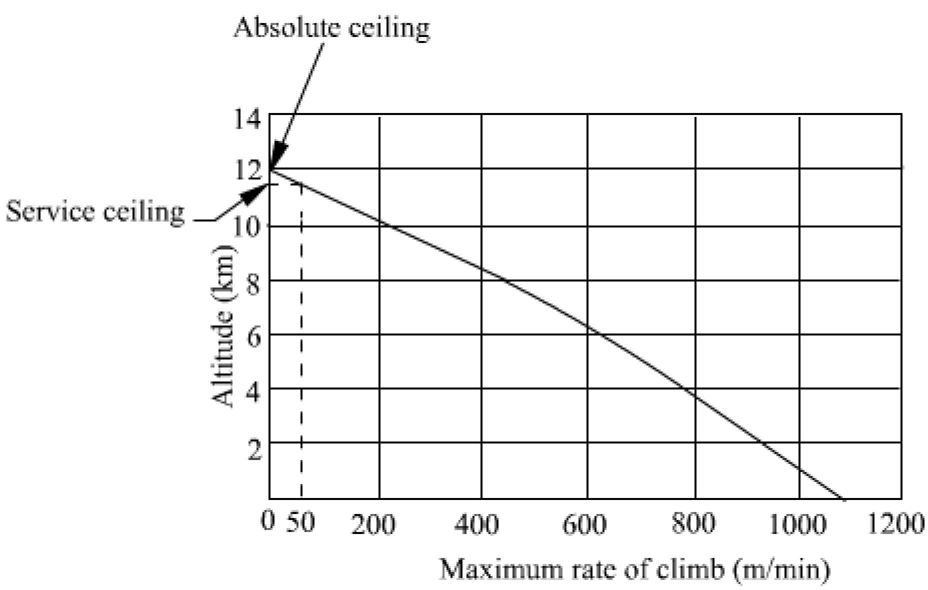


Variation of THP with flight speed at different altitudes - schematic

3. Define service and absolute ceilings.

Answer:

Typical variation of the maximum rate of climb, $(R/C)_{max}$, with altitude is shown below in the case of a jet airplane.



Variation of $(R/C)_{max}$ with altitude for a jet airplane

Absolute ceiling is the altitude at which $(R/C)_{max}$ is zero. Service ceiling is the altitude at which $(R/C)_{max}$ is 50m/min.

4. What are the conditions for maximum endurance of a jet powered airplane?

Answer:

During a flight the time elapsed in hrs (dE) when a quantity of fuel dW_f in Newtons (N) is consumed is:

$$dE = \frac{dW_f}{N \text{ of fuel/hr}}$$

For a jet airplane N of fuel/hr = T x TSFC also dW_f = - dW, the rate of change of W.

Further, $T = W\,(C_D / C_L)$

$$\text{Hence, } dE = \frac{-dW}{W \frac{C_D}{C_L} TSFC}$$

$$\text{Or } E = \int_{W_1}^{W_2} \frac{-dW}{W \frac{C_D}{C_L} TSFC}$$

W_1 = Weight at the start of the flight, W_2 = Weight at the end of the flight

Assuming C_L and TSFC to be constant :

$$E = \frac{C_L}{C_D} \frac{1}{TSFC} \ln\left(W_1/W_2\right)$$

Hence for maximum endurance, the flight should be at C_L corresponding to $(C_L/C_D)_{max}$.

5. Define neutral point.

Answer:

The location of c.g. of the airplane at which the airplane is neutrally stable, is called the neutral point. It is denoted by x_{NP}.As a brief explanation it may be added that $dC_m/d\alpha$ or $C_{m\alpha}$ is:

$$C_{m\alpha} = (C_{m\alpha})_{wing} + (C_{m\alpha})_{f,n,p} + (C_{m\alpha})_{h.tail}$$

$$(C_{m\alpha})_{wing} = \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right)$$

At neutral point $C_{m\alpha}$ is zero or

$$0 = \left(\frac{x_{NP}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + (C_{m\alpha})_{f,n,p} + (C_{m\alpha})_{h.tail}$$

or $\frac{x_{NP}}{\bar{c}} = \frac{x_{ac}}{\bar{c}} - (C_{m\alpha})_{f,n,p} - (C_{m\alpha})_{h.tail}$

6. What is the criterion for static longitudinal stability?

Answer:

The criterion for longitudinal static stability is as follows.

When an airplane is disturbed, by a small disturbance, in the plane of symmetry, it has a tendency to return to the equilibrium state.

In the equilibrium state M_{cg} is zero and the airplane flies at an angle of attack of α. M_{cg} is positive nose up, α is positive as shown in the figure below.

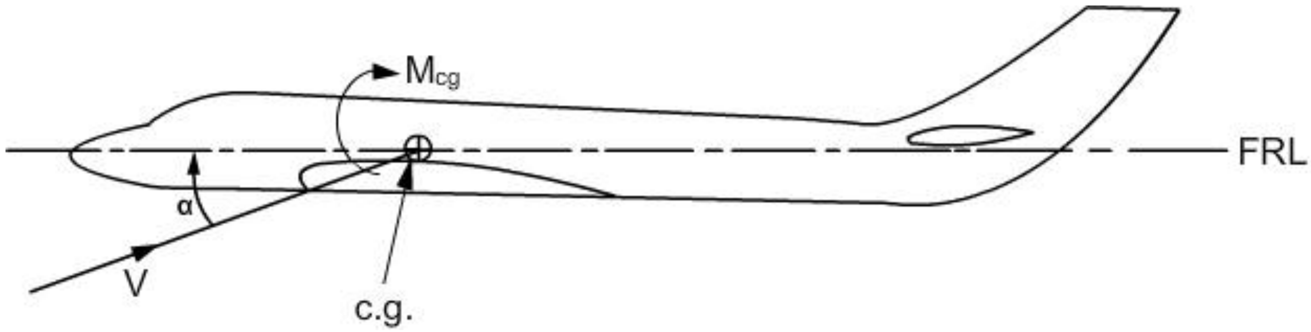
A disturbance changes α to $\alpha + \Delta\alpha$.

Thus for longitudinal static stability, a positive Δα should bring about negative M_{cg}

or $\frac{dM}{d\alpha}$ or $C_{m\alpha}$ < 0 for static stability

= 0 for neutral stability

> 0 for instability



Convention for M_{cg} and α

7.What is meant by dihedral effect?

Answer:

Rolling moment due to sideslip is called dihedral effect. The rolling moment coefficient is $C_l' = \dfrac{L'}{\frac{1}{2}\rho V^2 Sb}$, L' = rolling moment

The sideslip angle is denoted by 'β'. Hence, $dC_l'/d\beta$ or $C_{l\beta}'$ is the dihedral effect.

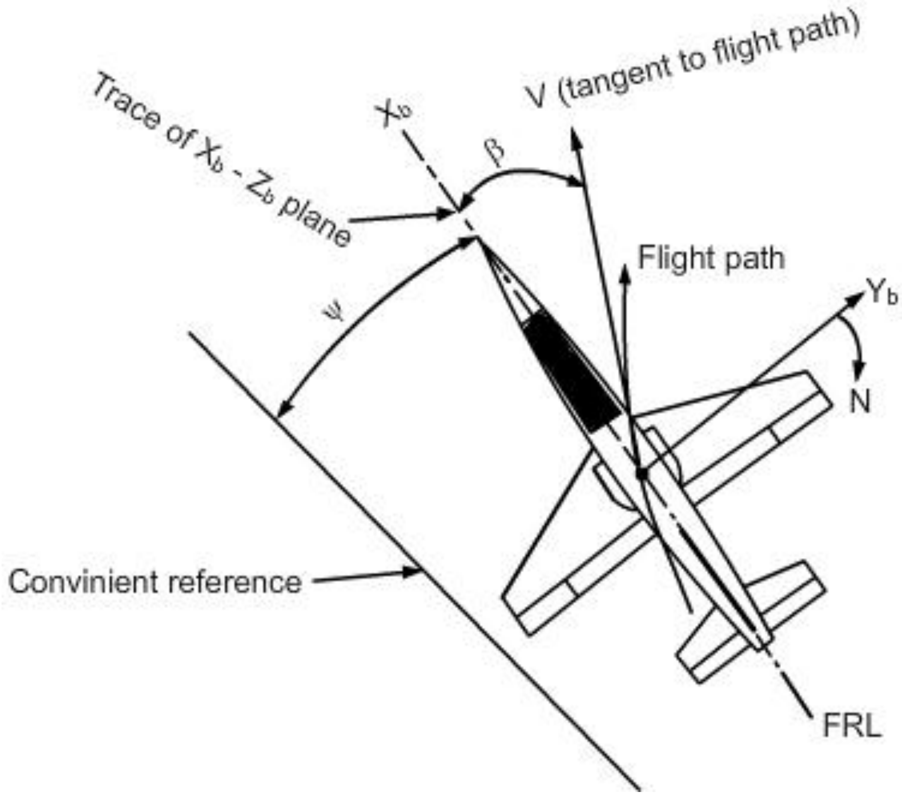
As a further explanation consider a airplane subjected to roll to right through angle ϕ . Due to the component WSin ϕ of the weight, airplane begins to sideslip to right or experiences a relative wind from right to left. This produces a positive β. Due to wing sweep, wing dihedral,fuselage, engine and vertical tail, an airplane with sideslip produces a rolling moment or has a dihedral effect.

8. Differentiate between yaw and sideslip angle.

Answer:

Sideslip is the angle between the plane of symmetry and the direction of motion. It is denoted by β and taken positive in the clockwise sense (Figure below). It may be pointed out that tangent to the flight path is the direction of motion.

Angle of yaw is the angular displacement of the airplane centre line, about a vertical axis, from a convenient horizontal reference line. It is denoted by Ψ and measured from the arbitrary chosen reference direction and taken positive in clockwise sense (Figure below).



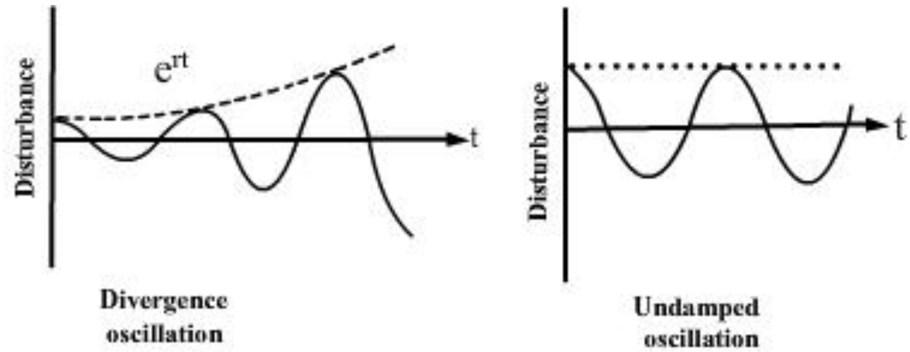
Sideslip and yaw

9. Graphically represent a system which is statistically stable but dynamically unstable.

Answer:

A system is said to be statically stable, when a small disturbance causes forces and moments that tend to move the system towards its undisturbed position. Whereas a system is said to be dynamically stable if it eventually returns to its original equilibrium position.

When a system, after disturbance, goes into a divergent oscillation or an undamped oscillation, it has only a tendency to return to equilibrium position but does not eventually return to it(Figures below). These are examples of systems that have static stability but no dynamic stability.



10. What is spiral divergence?

Answer:

The characteristic equation for lateral dynamic stability has five roots. A zero root, a large negative root, a small root which could be positive or negative and a pair of complex roots. The lightly damped real root is called spiral mode. When the root is positive i.e. unstable, the airplane looses altitude, gains speed, banks more and more with increasing turn rate. The flight path is a slowly tightening spiral motion. This motion is called spiral divergence.

Part B - (5 x 16 = 80 marks)

11a) An aircraft weighing 2,50,000 N has a wing area of 80 m^2 and its drag equation is $C_D = 0.016 + 0.04\, C_L^2$. Calculate (i) minimum thrust required (T_{min}) (ii) minimum power required (P_{min}) for straight and level flight and the corresponding true air speeds (V_{md} & V_{mp}) at sea level and at an altitude where $(\sigma)^{1/2} = 0.58$. Assume sea level air density to be 1.226 kg/m^3.

Answer:

The given data are :

W = 250,000 N, S = 80 m^2, $C_D = 0.016 + 0.04\, C_L^2$, $\rho_{S.L.} = 1.226$ kg/m^2

$$C_{Lmd} = \sqrt{\frac{C_{D0}}{K}} = \sqrt{\frac{0.016}{0.04}} = 0.6325$$

$$C_{Lmp} = \sqrt{3}\ \ C_{Lmd} = 1.0954$$

$$C_{Dmd} = 2\, C_{D0} = 0.032\,;\ C_{Dmp} = 4\, C_{D0} = 0.064$$

I) At sea level

$$T_{min} = W\frac{C_{Dmd}}{C_{Lmd}} = 250000 \times \frac{0.032}{0.6325} = 12648.2\ N$$

$$V_{md} = \sqrt{\frac{2W}{\rho S C_{Lmd}}} = \sqrt{\frac{2 \times 250000}{1.226 \times 80 \times 0.6325}} = 89.78\ m/s = 323.2\ kmph$$

$$V_{mp} = V_{md}/\sqrt[4]{3} = 68.22\ m/s = 245.6\ kmph$$

$$P_{min} = \frac{T_{mp}V_{mp}}{1000} = \frac{250000}{1000} \times \frac{0.064}{1.0954} \times 68.22 = 996.5\ KW$$

II) At h where $\sigma^{1/2} = 0.58$

$T_{min} = (T_{min})_{s.l.} = 12648.2$ N

$V_{md} = (V_{md})_{s.l.} / 0.58 = 154.79$ m/s = 557.3 kmph

$V_{mp} = (V_{mp})_{s.l.} / 0.58 = 117.62$ m/s = 423.4 kmph

$$P_{min} = (P_{min})_{s.l.}\ (V_{min})_h/(V_{min})_{s.l.} = 996.5 \times \frac{117.62}{68.22} = 1718.1\ KW$$

Results

The results are tabulated below

Quantity	h = s.l.	h where $\sqrt{\sigma} = 0.58$
T_{min}	12648.2 N	12648.2 N
P_{min}	996.5 KW	1718.1 KW
V_{md}	323.2 Kmph	557.3 Kmph
V_{mp}	245.6 Kmph	423.4 Kmph

12 (a) Write short notes on:

(i) International standard atmosphere.

(ii) Various types of drag of an airplane.

Answers:

i) International standard atmosphere (ISA)

A) Need for ISA and agency prescribing it

The properties of earth's atmosphere like pressure, temperature and density vary not only with height above the earth's surface but also with (a) the location on earth, (b) from day to day and (c) even during the day. However, the performance of an airplane is dependent on the physical properties of the earth's atmosphere. Hence, for the purposes of comparing (a) the performance of different airplanes and (b) the performance of the same airplane measured in flight tests on different days, a set of values for atmospheric properties have been agreed upon, which represent average conditions prevailing for most of the year, in Europe and North America. Though the agreed values do not represent the actual conditions anywhere at any given time, they are useful as a reference quantities. This set of values, called the International Standard Atmosphere (ISA), is prescribed by ICAO (International Civil Aviation Organization). This set is defined by the pressure and temperature at mean sea level, and the variation of temperature with altitude up to 32 km. With these values being prescribed, it is possible to find the required physical characteristics (pressure, temperature, density etc.) at any chosen altitude.

B) Features of ISA

The main features of the ISA are the standard sea level values and the variation of temperature with altitude. The air is assumed as dry perfect gas.

The standard sea level conditions are as follows:

Temperature (T_0) = 288.15 K = 15^0 C

Pressure (p_o) = 101325 N/m^2 = 760 mm of Hg

Rates of change of temperature in various regions are as follows:

- 6.5 K/km upto 11 km

0 K/km from 11 to 20 km

1 K/km from 20 to 32 km

The region of ISA from 0 to 11 km is referred to as troposphere. That between 11 to 20 km is the lower stratosphere and between 20 to 32 km is the middle stratosphere.

Note : Using the equation of state $p = \rho RT$, the sea level density (ρ_0) is obtained as 1.225 kg/m^3

(C) Variations of properties with altitude in ISA

For calculation of the variations of pressure, temperature and density with altitude, the following equations are used.

The equation of state $p = \rho RT$ (1)

The hydrostatic equation $dp/dh = -\rho g$ (2)

Substituting for ρ from the Eq.(1) in Eq.(2) gives:

$$dp / dh = -(p/RT)\, g$$

$$\text{or } (dp / p) = -g\, dh/RT \qquad (3)$$

Equation (3) is solved separately in troposphere and stratosphere, taking into account the temperature variations in each region. For example, in the troposphere, the variation of temperature with altitude is given by the equation

$$T = T_0 - \lambda h \qquad (4)$$

where T_0 is the sea level temperature, T is the temperature at the altitude h and λ is the temperature lapse rate.

Substituting from Eq.(4) in Eq.(3) gives:

$$(dp/p) = -\,g dh / R(T_0 - \lambda h) \qquad (5)$$

Equation (5) can be integrated between two altitudes h_1 and h_2. Taking h_1 as sea level and h_2 as the desired altitude (h), the integration gives the following equation

$$(p/p_0) = (T/T_0)^{(g/\lambda R)} \qquad (6)$$

where T is the temperature at the desired altitude (h) given by Eq.(4).

Equation (6) gives the variation of pressure with altitude.

The variation of density with altitude can be obtained using Eq.(6) and the equation of state. The resulting variation of density with temperature in the troposphere is given by:

$$(\rho/\rho_0) = (T/T_0)^{(g/\lambda R)-1} \qquad (7)$$

Thus both the pressure and density variations are obtained once the temperature variation is known.

As per the ISA, R = 287.05287 m^2 sec^{-2} K and g = 9.80665 m/s^2.

Using these and λ = 0.0065 K/m in the troposphere, yields ($g/R\lambda$) as 5.25588.

Thus, in the troposphere, the pressure and density variations are :

$$(p/p_0) = (T/T_0)^{5.25588} \qquad (8)$$

$$(\rho/\rho_0) = (T/T_0)^{4.25588} \qquad (9)$$

It may be noted that

T = 288.15 – 0.0065 h; h in m and T in K.

Similar expressions can be obtained for other regions of ISA.

(ii)Various types of drag of an airplane

Answer:

The drag of an airplane can be expressed as

$D = D_{wing} + D_{fuselage} + D_{nacelle} + D_{h.tail} + D_{v.tail} + D_{Landing\ gear} + D_{other} + D_{int}$

Or $C_D = C_{DW} + C_{Df} + C_{Dn} + C_{Dht} + C_{Dvt} + C_{Dlg} + C_{Dother} + C_{Dint}$

where D_{other} = Drag of other items like bombs etc.

D_{int} = interference drag

A brief discussion of various types of drags is as follows.

Skin friction drag, pressure drag, profile drag, induced drag and drag coefficient of wing

The drag coefficient of a wing consist of the (i) the profile drag due to airfoil (C_d) and (ii) the induced drag due to the finite aspect ratio of the wing (C_{Di}). The symbols c_d and c_l refer to drag coefficient and lift coefficient of the airfoil. The profile drag of the airfoil consists of the skin friction drag and the pressure drag. It may be added that an element on the surface of an airfoil, kept in a flow, experiences shear stress (τ) tangential to the surface and pressure (p) normal to it (as shown in figure below). The shear stress multiplied by the area of the element gives the tangential force. The component of this tangential force in the free stream direction when integrated over the profile gives the skin friction drag. Similarly the pressure distribution results in normal force on the element whose component in the free stream direction, integrated over the profile gives the pressure drag. The pressure drag is also called form drag. The sum of the skin friction drag and the pressure drag is called profile drag. The profile drag depends on the airfoil shape, Reynolds number, angle of attack and surface roughness.

Shear stress (τ) and pressure (p) on an airfoil

As regards the induced drag, it may be mentioned that a wing has a finite span. This result in a system of trailing vortices and the induced angle due to these vortices tilts the aerodynamic force rearwards. This results in a component in the free stream direction which is called induced drag. The induced drag coefficient is given by :

$$C_{Di} = \frac{C_L^2}{\pi A}(1+\delta)$$

Where A is the wing aspect ratio and δ is a factor which depends on wing aspect ratio, taper ratio and sweep.

The drag of the wing is the sum of the profile drag and induced drag.

A similar procedure can be used to estimate drags of horizontal and vertical tails.

Drag coefficient of fuselage

The drag coefficient of a fuselage (C_{Df}) consists of the drag of the fuselage at zero angle of attack $(C_{Do})_f$ plus drag due to angle of attack. It can be expressed as :

$$C_{Df} = (C_{Do})_f + K(\alpha)^2$$

For a streamlined body $(C_{D0})_f$ is mainly skin friction drag and depends on (i) Reynolds number, based on length of fuselage (l_f), (ii) surface roughness and (iii) fineness ratio of fuselage.

For a bluff body, C_{D0f} is mainly pressure drag.

The drag coefficients of other bodies like engine nacelle, external fuel tanks and bombs can also be estimated in a similar manner.

Drag coefficients of other components

The drag coefficient of other components like landing gear is mainly pressure drag and is based on areas specific to the component.

Intereference drag

While presenting the data on the drag of wing or fuselage or any other component of the airplane, the data generally refers to the drag of that component when it is alone in the airstream and free from the influence of any other component. Whereas, in an airplane, the wing, the fuselage and the tails are present in close proximity of each other and the flow past one component is influenced by the others. As a result, the drag of the airplane, as a combination of different components, is generally is more than the sum of the drags of individual components. The additional drag due to interference is called interference drag. It is generally between 2 to 10% of the sum of the drags of all components.

Parabolic drag polar, parasite drag and induced drag

The drag polar obtained by adding the drag coefficients of individual components can be approximated as :

$$C_D = C_{D0} + KC_L^2$$

C_{Do} is called zero lift drag coefficient or parasite drag coefficient

The term KC_L^2 is called induced drag coefficient or more appropriately lift dependent drag coefficient. K is written as:

$$K = \frac{1}{\pi Ae}$$

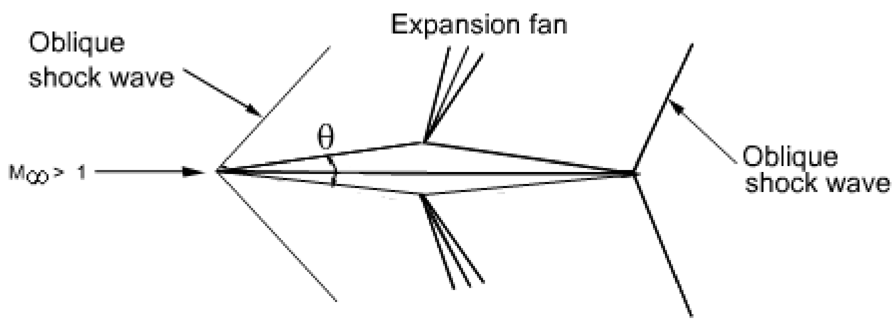
e is called Oswald efficiency factor.

Wave drag

This is a pressure drag which occurs at transonic and supersonic Mach numbers. Its origin can be briefly described as follows.

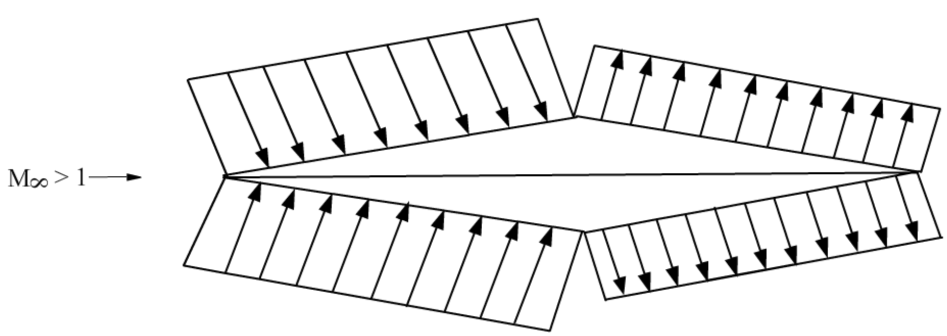
When a supersonic flow encounters a convex corner a shock wave is produced and when it encounters a concave corner an expansion fan is produced. Pressure increases behind a shock wave and it decreases along an expansion fan.

A typical pattern of shock wave and expansion fan in a flow past a diamond airfoil is shown below.



Shock waves and expansion fan on a diamond airfoil at α = 0

A typical distribution of pressure coefficient (C_p) is shown below.



C_p distribution on a diamond airfoil at α = 0

It is evident that this pressure distribution would produce a drag. It is called wave drag. Wave drag depends on the shape of airfoil, thickness ratio and angle of attack or lift coefficient. In case of a wing the wave drag would be modified by aspect ratio.

It is noticed that even at transonic and supersonic Mach numbers C_D of the airplane can be expressed as:

$$C_D = C_{D0} + K C_L^2$$

but in this case C_{D0} and K vary with Mach number.

13 (b) Discuss in detail the power effects on static longitudinal stability for a jet powered airplane.

Answer:

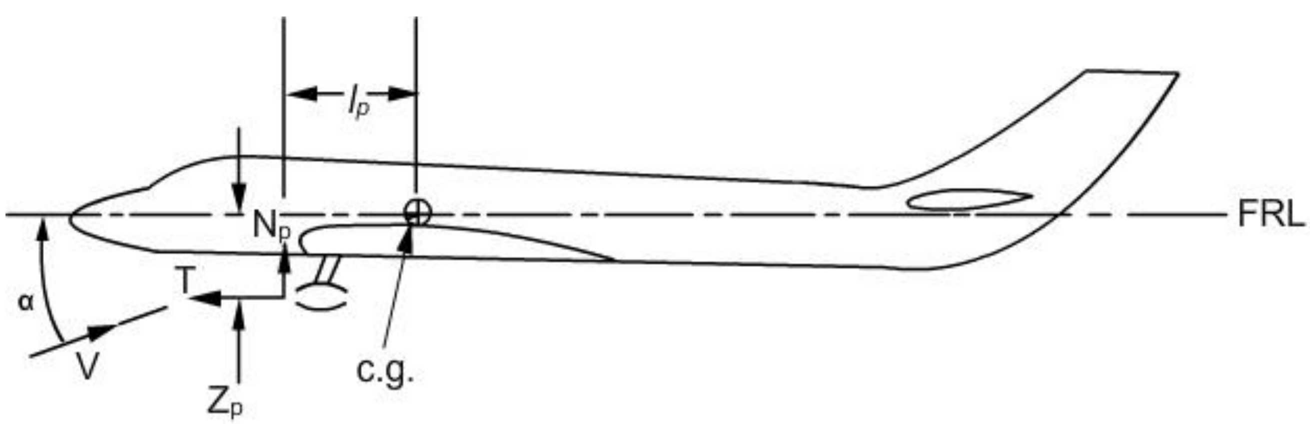
The contributions of power plant to C_{mcg} and $C_{m\alpha}$ have two aspects namely direct contribution and indirect contribution. The direct contribution appears when the direction of the thrust vector does not coincide with the line passing through the c.g.(figure below). The direct contribution is written as:

$$M_{cgp} = T \times Z_p \qquad (1)$$

where T is the thrust and Z_p is the perpendicular distance of thrust line from FRL; positive when c.g. is above thrust line.

In non-dimensional from Eq.(1) is expressed as:

$$C_{mcgp} = M_{cgp} \,/\, (\frac{1}{2}\,\rho\, V^2\, S\, \bar{c}) \qquad (2)$$



Contribution of thrust to C_{mcg}

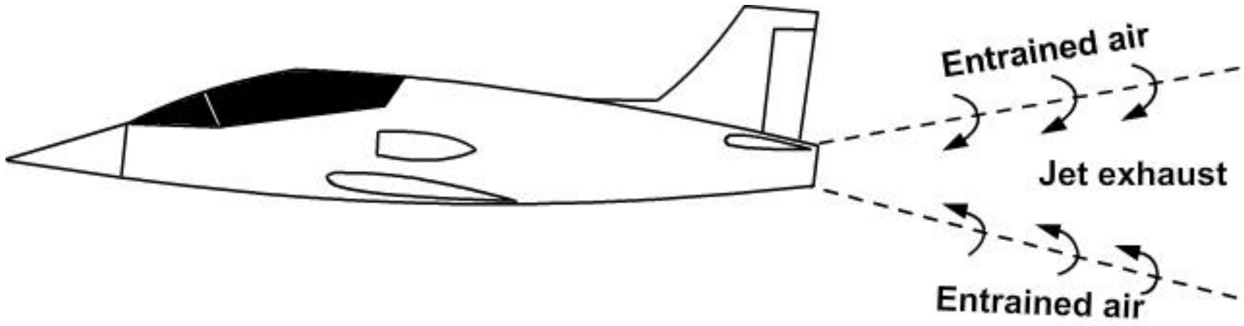
The thrust required varies with flight speed and altitude. Hence C_{mcgp} would vary with flight condition. However the thrust setting does not change during the disturbance and hence there is no contribution to $C_{m\alpha}$

The contribution to $C_{m\alpha}$ comes from another cause. Consider a jet engine at an angle of attack as shown in the figure above. The free stream velocity (V) is at an angle (α) to the axis of the engine. As the airstream passes through the engine it leaves in a nearly axial direction. This change of direction results in a normal force (N_p) in addition to the thrust (T). N_p acts at distance l_p from the c.g. and hence produces a moment N_p x l_p. The value of N_p depends on the angle of attack of the airplane and hence the term $N_p l_p$ depends on α. This will contribute to $C_{m\alpha}$.

It is evident from the above figure that when the engine is ahead of c.g., the contribution to $C_{m\alpha}$ due to normal force would be positive or destabilizing. If the engine is near the rear of the airplane, the contribution of normal force to $C_{m\alpha}$ will be negative and hence stabilizing.

Indirect contribution of power plant:

In the case of an airplane with jet engine the exhaust expands in size as it moves downwards and entrains the surrounding air (see figure below). This would induce an angle to the flow; the induced angle would be positive in the region below the jet and negative in the region above the jet. In military airplanes the engine exhaust comes out from the rear of the fuselage and would affect the horizontal tail, generally located above the rear fuselage, by inducing a downwash in addition to that due to wing (figure below). This effect will also come into picture in case of passenger airplanes with rear mounted engines. To alleviate this, the horizontal tail is sometimes mounted above the vertical tail.



Effect of jet exhaust

Remarks:

i) The contribution of engine depends also on the engine power setting which in turn depends on flight condition or C_L. Hence the level of stability $(C_{m\alpha})$ will depend on C_L and also will be different when engine is off or on.

ii) It is difficult to accurately estimate the effects of power on $C_{m\alpha}$. A rough estimate would be $(dC_m/dC_L)_p = 0.04$ or $C_{m\alpha p} = 0.04\, C_{L\alpha}$

14 (b) Discuss briefly the following:

(i) Basic requirements of the rudder.

(ii) Aileron reversal

(iii) Adverse yaw.

Answers:

(i) Basic requirements of rudder

Control of rotation of the airplane about the z-axis is provided by the rudder. The critical conditions for the design of the rudder are:

(A) adverse yaw,
(B) cross wind take-off and landing,
(C) asymmetric power for multi-engined airplane and
(D) spin

Adverse yaw and its control

When an airplane is rolled to the right, the rate of roll produces a yawing moment tending to turn the airplane to the left. Similarly a roll to left produces yaw to right. Hence, the yawing moment produced as a result of the rate of roll is called adverse yaw. The production of adverse yaw can be explained as follows. Consider an airplane rolled to right, i.e. right wing down. Let the rate of roll be ‘p’. The rate of roll has the following two effects.

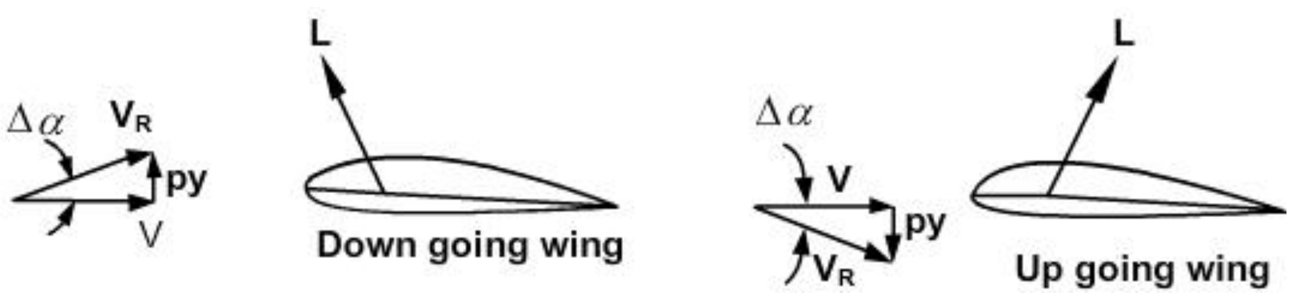
a) A roll to right implies less lift on the right wing and more lift on the left wing. This is brought about by aileron deflection – in the present case an up aileron on the right wing and a down aileron on the left wing. Since, C_L on the right wing is less than C_L on the left wing, the induced drag coefficient (C_{Di}) on the right wing is less than C_{Di} on the left wing. This results in a yawing moment causing the airplane to yaw to left.

b) Due to the rolling velocity (p) a section on the down going wing at a distance ‘y’ from the FRL experiences a relative upward wind of magnitude ‘py’. At the same time a section on the up going wing experience a relative downward velocity of magnitude ‘py’.This results in the change of direction of the resultant velocity on the two wing halves (Figure below). Now the lift vector, being perpendicular to the resultant velocity, is bent forward on the down going wing and bent backwards on the up going wing. Consequently, the horizontal components of the lift on the two wing halves produce a moment tending to yaw the airplane to left.

A rough estimate of effect of adverse yaw is

$$(C_n)_{adverseyaw} \approx -\frac{C_L}{8}\frac{pb}{2V}$$

where, p = rate of roll in radians per second; b = wing span and
V = flight velocity.



Effect of rate of roll

An airplane is generally designed for a specific value of (pb/2V). For example

pb / 2V = 0.07 for cargo/bomber

pb / 2V = 0.09 for fighter

Hence, one of the criteria for rudder design is that it must be powerful enough to counter the adverse yaw at the prescribed rate of roll.

Control in cross wind take-off and landing

When there is a cross wind, the effect is as if the airplane is sideslipping; $\Delta\beta = v/V$, where v is the cross wind velocity. Further the tendency of an airplane possessing directional static stability is to align itself with the wind direction. During take-off and landing the pilot has to keep the airplane along the runway. Hence, another criterion for the design of the rudder is that it must be able to counteract the yawing moment due to sideslip produced by cross wind $(C_{n\beta}\Delta\beta)$.This criterion becomes more critical at lower speeds because (a) the effectiveness of the rudder, being proportional to V^2, is less at lower flight speeds and (b) $\Delta\beta$ being proportional to 1/V, is high at low flight speeds.

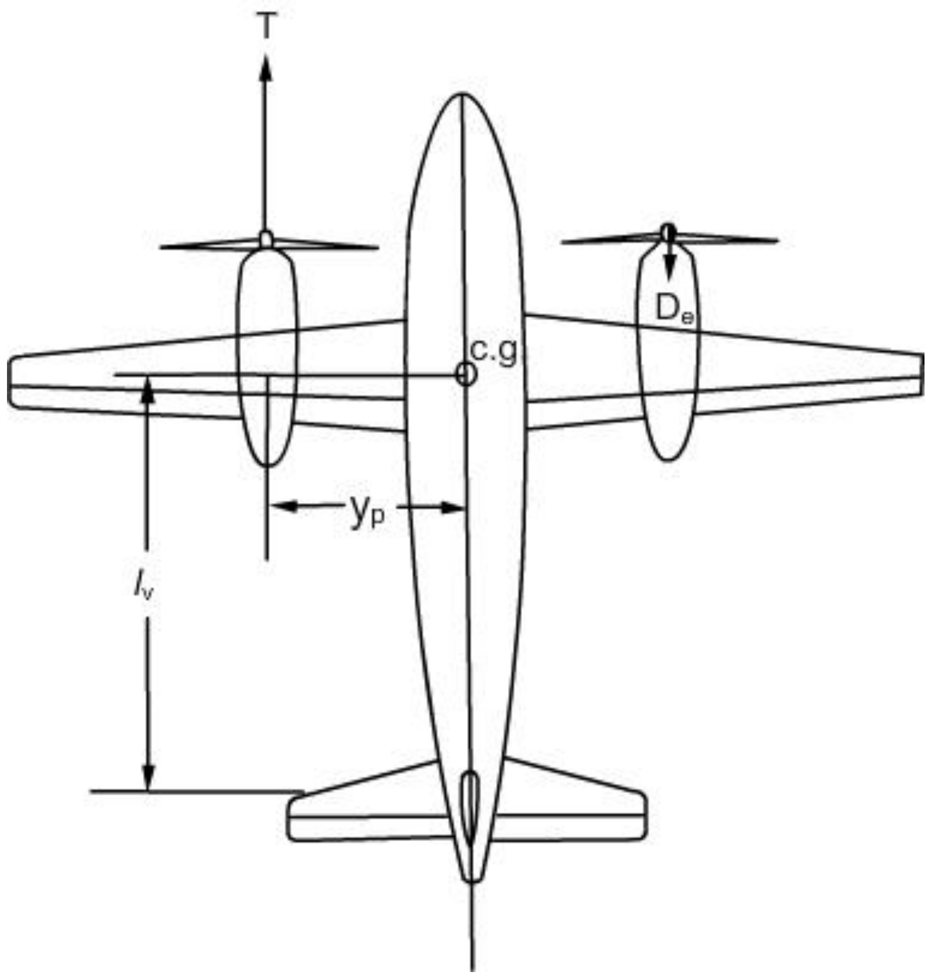
Generally the rudder must be able to overcome v = 51 ft/s or 15 m/s at the minimum speed for the airplane.

Control in asymmetric power

Consider a multi-engined airplane. When one of the engines fails the following changes take place.

(a) The engine that is operating causes a yawing moment T x y_p (Figure below)

(b) In the case of engine propeller combination the drag (D_e) of the propeller will be large if it is held in the stopped condition. Hence generally the propeller is feathered so that it does wind milling.



Airplane with one engine – off

In this situation there is drag but it is small. In the case of airplane with jet engines, the failed engine is held in idling condition. The drag due to failed engine causes a yawing moment which reinforces the yawing moment due to the operating engine. If the engine on the right wing has failed then the yawing moment due to the operating and the failed engines would cause a positive yawing moment.(Figure above)

$$N_e = \Delta T \times y_p \tag{1}$$

Where $\Delta T =$ thrust of live engine + drag of dead engine.

$$C_{ne} = \frac{N_e}{\frac{1}{2}\rho V^2 Sb} \tag{2}$$

The yawing moment due to engine is controlled by the rudder deflection.

The maximum yawing moment coefficient due to the rudder would be $C_{n\delta r}(\delta_r)_{max}$. This remains almost constant with speed. However, form Eq.(2) it is seen that the yawing moment due to engine (C_{ne}) increases as flight speed (V) decreases. These facts viz. $(C_{n\delta}(\delta_r)_{max})$ being constant and C_{ne} increasing as V decreases, indicate that there is a speed (V_{mc}) below which the full rudder deflection $(\delta_r)_{max}$ would not be able to control the airplane in the event of engine failure. This speed is referred to as minimum control speed (V_{mc}).This must be lower than or equal to the desired minimum speed of the airplane.

Control for spin recovery

Spin is a flight condition in which the airplane wings are stalled and it moves downward rapidly along a helical path. The only control that is still effective is the rudder. The way to come out of the spin is to stop the rotation, go into a dive and pull out. The rudder must be powerful enough to get the airplane out of spin.

14 b (ii) Aileron reversal

In stability analysis, it is generally assumed that the airplane is rigid. However, it is elastic. It (airplane) deflects and twists under loads.

When an aileron is deflected down it increases the lift on that wing half but, it also makes C_{mac} more negative. Consequently the wing twists and the angle of attack decreases. The amount of twist depends on (a) the change in pitching moment (ΔM_{ac}), (b) wing stiffness and (c) wing geometric parameters like aspect ratio, taper ratio and sweep.

Let, the aileron deflection be $\Delta\delta_a$. This brings about a change ΔC_{mac}. Now ΔC_{mac} brings about a moment $\Delta M_{ac} = \frac{1}{2}\rho V^2 S \bar{c} \Delta C_{mac}$. It is seen that ΔM_{ac} depends on V^2. Thus, the twisting moment and hence the twist increases with flight speed. There is a speed, called aileron reversal speed, at which the reduction in the angle of attack due to twist will nullify the increase in the lift due to deflection of aileron. Beyond this speed a downward deployment of aileron would actually decrease lift. This is called aileron reversal. To retain the roll control, the high speed airplanes have spoiler ailerons. When a spoiler is deployed, it reduces lift on that wing half and the wing rolls.

14 b (iii) Adverse yaw

A description of this phenomenon has already been given in answer to question 14 (b) i.

15 (a) discuss the following:

(i) Phugoid motion.

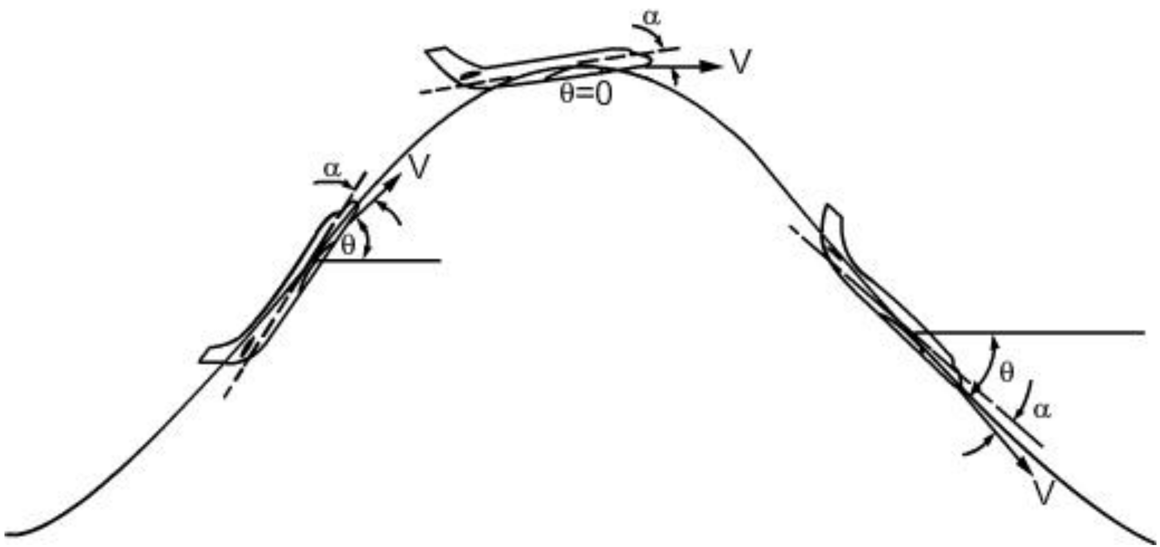
(ii) Stability derivatives in longitudinal dynamics.

Answers:

i) Phugoid motion

The characteristic equation for the longitudinal motion generally has two pairs of complex roots. One of these has a short period and is heavily damped. The motion corresponding to this root is called short period oscillation. This mode lasts for the first few seconds during which the angle of attack changes rapidly and attains the undisturbed value. The motion corresponding to the second root has a long period and low damping. This mode is called long period oscillation (LPO). It is also called Phugoid. Because of low damping it takes long time to subside. Even if the mode is unstable the pilot has enough time to take corrective action. The damping is proportional to C_D and a streamlined airplane will have lower damping. The period is proportional to the undisturbed velocity (u_0) or the period will be longer at higher flight speeds.

During the phugoid the angle of attack remains roughly constant but the angle of pitch ($\Delta\theta$) and velocity change periodically. This implies that the altitude of the airplane also changes periodically (Figure below). Noting that the damping of phugoid is very light and the flight speed is changing periodically, the motion during a cycle can be considered as a flight with total energy of the airplane (i.e. sum of potential energy and kinetic energy) remaining nearly constant. As the airplane goes through a cycle there is exchange between potential and kinetic energy of the airplane. From the figure below it is noted that, as the airplane climbs it looses kinetic energy and gains potential energy. At the crest the speed is minimum. As it descends the speed increases and altitude decreases. At the trough the velocity is maximum.



Motion of airplane during phugoid

15 a (ii) Stability derivatives in longitudinal dynamics

Based on Newton's second law the equations of motion for the longitudinal motion in standard notation can be written as:

$$m(\dot{u} + qw - rv) = X - mg\sin\theta_0$$

$$m(\dot{w} + pv - qu) = Z + mg\cos\theta_0$$

$$I_{yy}\dot{q} + (I_{xx} - I_{zz})rp + I_{xz}(p^2 - r^2) = M$$

When small perturbation analysis is used the quantities u,v,w,p,q,r,X,Z and M in the above equations are replaced by $\left(u_0 + \Delta u\right)$, Δv, Δw, Δq, Δr, ΔX, ΔZ and ΔM.

Ignoring the product terms in the resulting equations yields :

$m\,\Delta\dot{u} = \Delta X - mg\cos\theta_0\ \Delta\theta$

$m(\Delta\dot{w} - u_0\Delta q) = \Delta Z - mg\sin\theta_0\ \Delta\theta$

$I_{yy}\,\dot{q} = \Delta M$

where ΔX, ΔZ and ΔM are the changes in aerodynamic and propulsive forces and moments during the disturbed motion. These quantities are functions of Δu, Δv, Δw, Δp, Δq, Δr, $\Delta\dot{u}$, $\Delta\dot{v}$, $\Delta\dot{w}$, $\Delta\delta_e$, $\Delta\delta_a$, $\Delta\delta_e$ etc.

Consider ΔX.It can be expressed as

$$\Delta X = \frac{\partial X}{\partial u}\Delta u + \frac{\partial X}{\partial v}\partial v + ... + \frac{\partial X}{\partial \dot{u}}\Delta\dot{u} + ... \frac{\partial X}{\partial \delta_e}\Delta\delta_e$$

However, to avoid unnecessary complications, ΔX, ΔZ etc. are taken to depend on only a few relevant quantities i.e.

$$\Delta X = \frac{\partial X}{\partial u}\Delta u + \frac{\partial X}{\partial w}\Delta w + \frac{\partial X}{\partial \delta_e}\Delta\delta_e$$

$$\Delta Z = \frac{\partial Z}{\partial u}\Delta u + \frac{\partial Z}{\partial w}\Delta w + \frac{\partial Z}{\partial \dot{w}}\Delta\dot{w} + \frac{\partial Z}{\partial q}\Delta q + \frac{\partial Z}{\partial \delta_e}\Delta\delta_e$$

$$\Delta M = \frac{\partial M}{\partial u}\Delta u + \frac{\partial M}{\partial w}\Delta w + \frac{\partial M}{\partial \dot{w}}\Delta\dot{w} + \frac{\partial M}{\partial q}\Delta q + \frac{\partial M}{\partial \delta_e}\Delta\delta_e$$

The quantities $\frac{\partial X}{\partial u}, \frac{\partial X}{\partial w}, \ldots\ldots\ldots, \frac{\partial M}{\partial \delta_e}$ are called stability derivatives. Expressions can be derived for each of these quantities. The procedure is illustrated for $\partial X/\partial u$.

$$\Delta X = -\Delta D + \Delta T$$

Hence, $\frac{\partial X}{\partial u} = -\frac{\partial D}{\partial u} + \frac{\partial T}{\partial u} = -\frac{\partial}{\partial u}(\frac{1}{2}\rho u^2 S C_D) + \frac{\partial T}{\partial u} = -\frac{1}{2}\rho S\ (u_0^2\frac{\partial C_D}{\partial u} + 2u_0 C_D) + \frac{\partial T}{\partial u}$

$\frac{\partial C_D}{\partial u}$ can be written as :

$\frac{\partial C_D}{a_0\,\partial(u/a_0)} = a_0\frac{\partial C_D}{\partial M_1}$, a_0 = speed of sound and $M_1 = \frac{u_0}{a_0}$

$\frac{\partial C_D}{\partial M_1} = 0$ at subcritical speeds.

At transonic and supersonic speed it can be evaluated from variation of C_D with Mach number.

$\frac{\partial \mathbf{T}}{\partial \mathbf{u}}$:

For jet engine T may be roughly constant with u and $\partial T / \partial u = 0$. For piston engined airplane T = THP/u and $\frac{\partial T}{\partial u} = \frac{-THP}{u^2} = -\frac{D}{u}$

Note:

In certain texts on stability analysis, the expression for $\frac{\partial X}{\partial u}$ may be written in a slightly different form.